

# Anti-symmetric rank-two Tensor Matter Field on Superspace for $N_T = 2$

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## Abstract

In this work, we discuss the interaction between anti-symmetric rank-two tensor matter and topological Yang-Mills fields. The matter field considered here is the rank-2 Avdeev-Chizhov tensor matter field in a suitably extended  $N_T = 2$  SUSY. We start off from the  $N_T = 2$ ,  $D = 4$  superspace formulation and we go over to Riemannian manifolds. The matter field is coupled to the topological Yang-Mills field. We show that both actions are obtained as  $Q$ -exact forms, which allows us to write the energy-momentum tensor as  $Q$ -exact observables.

## 1 Introduction

Topological field theories such as Chern-Simons and BF-type gauge theories probe space-time in its global structure, and this aspect has a significative relevance in quantum field theories. On the other hand, there is great deal of interest in anti-symmetric rank-2 tensor fields that can be put into two categories: gauge fields or matter fields. In recent years, Avdeev Chizhov [1, 2, 3] proposed a model where the antisymmetric tensor behaves as a matter field.

In a recent work [4], Geyer-Mülsch presented a formulation until then unknown in the literature, which is a construction of the Avdeev-Chizhov action described in the topological formalism [5]. This was built for  $N_T = 1$  and generalized for  $N_T = 2$ . Known the properties of the anti-symmetric rank-two tensor matter field theory, also called Avdeev-Chizhov field [6], the supersymmetric properties and characteristics are presented also in ref. [7]; following this formalism, we shall write this action in the superfield formalism, as presented by Horne [8] in topological theories as a Donaldson-Witten topological theories [9, 5].

Our goal in this work is to discuss the interaction between matter and topological Yang-Mills fields as presented by Geyer-Mülsch [4] for  $N_T = 1$  and  $N_T = 2$ . The matter field considered here is the rank-2 tensor matter field as a complex self-duality condition [6]. Thus, we write this field now as an anti-symmetric rank-two tensor matter superfield in  $N_T = 2$  SUSY in the superspace formalism, founded also in [7]. The matter field is coupled to the topological Yang-Mills connection by means of the Blau-Thompson action. We write the Yang-Mills superconnection

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as a 2–superform in a superspace with four bosonic dimensions spacetime described by Grassmann-odd coordinates and two fermionic dimensions described by Grassmann-even coordinates, and then construct the action in a superfield formalism following the definitions by Horne [8]. Then, we go over to Riemannian manifolds duely described in terms of the vierbein and the spin connection, where we take the gravitation as a background. We introduce and discuss the Wess-Zumino gauge condition induced by the shift supersymmetry better detailed in [10]. Then, we arrive at a topological invariant action as the sum of the Avdeev-Chizhov’s action coupled to the topological super-Yang-Mills action; both actions are obtained as  $Q$ –exact forms, and the energy-momentum tensor is shown to be  $Q$ –exact.

## 2 The $N_T = 2$ Super-conection, Super-curvature and Shift Algebra

Let us now consider the Donaldson-Witten theory, whose space of solutions is the space of self-dual instantons,  $F = *F$ . To follow our superfield formulation, we shall proceed with the definition of the action of Horne [8] and Blau-Thompson [13, 14]. The  $N_T = 2$  superfield conventions are the ones of [10]. The superfields superconnection and its associated superghosts are given as below:

$$\hat{A} = \hat{A}^a T_a, \quad \hat{C} = \hat{C}^a T_a, \quad (2.1)$$

whose the generators belonging the Lie algebra:

$$[T_a, T_b] = i f_{ab}{}^c T_c. \quad (2.2)$$

Expanding the superforms (2.1) in component superfields, we have

$$\hat{A} = A(x_\mu, \theta^I) + E_I(x_\mu, \theta^I) d\theta^I, \quad \hat{C} = C(x_\mu, \theta^I), \quad (2.3)$$

with  $I = 1, 2$ ; in component fields, it comes out as below:

$$A(x, \theta) = a(x) + \theta^I \psi_I(x) + \frac{1}{2} \theta^2 \alpha(x), \quad (2.4)$$

$$E_I(x, \theta) = \chi_I(x) + \theta^I \phi_{IJ}(x) + \frac{1}{2} \theta^2 \eta_I(x), \quad (2.5)$$

$$C(x, \theta) = c(x) + \theta^I c_I(x) + \frac{1}{2} \theta^2 c_F(x). \quad (2.6)$$

The associated supercurvature is defined as

$$\hat{F} = \hat{d}\hat{A} + \hat{A}^2 = (dA + A^2) + (\partial_I A + D_A E_I) d\theta^I + \frac{1}{2} (\partial_I E_J + \partial_J E_I + [E_I, E_J]) d\theta^I d\theta^J, \quad (2.7)$$

which can also be expressed as:  $\hat{F} = F + \Psi_I d\theta^I + \Phi_{IJ} d\theta^I d\theta^J$ , whose components read as follows:

$$F = f - \theta^I D_a \psi_I + \frac{1}{2} \theta^2 (D_a \alpha + \frac{1}{2} \varepsilon^{IJ} [\psi_I, \psi_J]), \quad (2.8)$$

$$\begin{aligned} \Psi_I &= \psi_I + D_a \chi_I + \theta^J (\varepsilon_{IJ} \alpha - \theta^J D_a \phi_{IJ} + \theta^J [\psi_J, \chi_I]) \\ &\quad + \theta^2 (\frac{1}{2} D_a \eta_I - \frac{1}{2} \varepsilon^{KJ} [\psi_K, \phi_{IJ}] + \frac{1}{2} [\alpha, \chi_I]), \end{aligned} \quad (2.9)$$

$$\begin{aligned} \Phi_{IJ} &= \frac{1}{2} \{ \phi_{IJ} + \phi_{JI} + [\chi_I, \chi_J] + \theta^K (\varepsilon_{KI} \eta_J + \varepsilon_{JK} \eta_I + [\chi_I, \phi_{JK}] + [\phi_{IK}, \chi_J]) \\ &\quad + \frac{1}{2} \theta^2 ([\chi_I, \eta_J] + [\eta_I, \chi_J] - \varepsilon^{KL} [\phi_{IK}, \phi_{JL}]) \}, \end{aligned} \quad (2.10)$$

where  $f = da + a^2$  and the covariant derivatives in  $a$  being given by  $D_a(\cdot) = d(\cdot) + [a, (\cdot)]$ ; the symbol  $(\cdot)$  represents any field which the derivative act upon. This formalism with  $N_T = 2$ , it can be found as an example in the work [11].

The SUSY number,  $s$ , is defined by attributing  $-1$  to  $\theta$ . Thus, the supersymmetry generators,  $Q$ , have  $s = 1$ . The BRST transformation of the superconnection (2.3) is  $s\hat{A} = -\hat{d}\hat{C} - [\hat{A}, \hat{C}] = -\hat{D}_{\hat{A}}\hat{C}$  and component superfields, is given by

$$\begin{aligned} sA &= -dC - [A, C] = -D_A C, \\ sE_I &= -\partial_I C - [E_I, C] = -D_I C, \\ sC &= -C^2, \end{aligned} \quad (2.11)$$

which in components take the form:

$$\begin{aligned} sa &= -dc - [a, c] = -D_a c, \\ s\psi_I &= -[c, \psi_I] - D_a c_I, \\ s\alpha &= -[c, \alpha] - D_a c_F + \varepsilon^{IJ} [c_I, \psi_J], \\ s\chi_I &= -[c, \chi_I] - c_I, \\ s\phi_{IJ} &= -[c, \phi_{IJ}] - \varepsilon_{IJ} c_F + [\chi_I, c_J], \\ s\eta_I &= -[c, \eta_I] - [c_F, \chi_I] + \varepsilon^{JK} [c_J, \phi_{IK}], \\ sc &= -c^2, \\ sc_I &= -[c, c_I], \\ sc_F &= -[c, c_F] + \frac{1}{2}\varepsilon^{IJ} [c_I, c_J]. \end{aligned} \quad (2.12)$$

and the super-covariant derivative is decomposed as:  $\hat{D}_{\hat{A}} = D_A + d\theta^I D_I$ .

The supersymmetry transformations or shift symmetry transformations are defined as:

$$Q_I A = \partial_I A, \quad Q_I E_J = \partial_I E_J, \quad Q_I C = \partial_I C;$$

in components, they read as follows:

$$\begin{aligned} Q_I a &= \psi_I, & Q_I \psi_J &= -\varepsilon_{IJ} \alpha, & Q_I \alpha &= 0, \\ Q_I \chi_J &= \phi_{JI}, & Q_I \phi_{JK} &= -\varepsilon_{IK} \eta_J, & Q_I \eta_J &= 0, \\ Q_I c &= c_I, & Q_I c_I &= -\varepsilon_{IJ} c_F, & Q_I c_F &= 0. \end{aligned} \quad (2.13)$$

Next, we believe it is interesting to introduce and discuss a sort of Wess-Zumino gauge choice associated to the shift symmetry above, which is the topological BRST transformation. The Wess-Zumino <sup>2</sup> gauge seen in [12, 10], is here defined by the condition

$$\chi_I = 0 \quad \text{and} \quad \phi_{[IJ]} = 0, \quad (2.14)$$

due to the linear shift in the transformations (2.12) for scalar fields  $\chi_I$  and  $\phi_{IJ}$  respectively, with parameters given by the ghost fields,  $c_I$  and  $c_F$ . There exists now, only the symmetric field  $\phi_{(IJ)}$ , that we write from now on simply as  $\phi_{IJ}$ . This condition is not SUSY-invariant under  $Q_I$ , and it can be defined in terms of the infinitesimal fermionic parameter  $\epsilon^I$  as

$$\tilde{Q} = \epsilon^I \tilde{Q}_I.$$

This operator leaves the conditions (2.14) invariant, and it is built up by the combinations of  $Q$  with the BRST transformations in the Wess-Zumino gauge, such that

$$\tilde{Q} = (s + Q)|_{c_I = \epsilon^J \phi_{IJ}, \quad c_F = \frac{1}{2}\epsilon^J \eta_J}. \quad (2.15)$$

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<sup>2</sup>This name is given since we are dealing with a linear gauge and scalar ghost field.

The results in terms of component fields are displayed below:

$$\begin{aligned}
\tilde{Q}a &= -D_a c + \epsilon^I \psi_I, \\
\tilde{Q}\psi_I &= -[c, \psi_I] - \epsilon^J D_a \phi_{IJ} + \epsilon_I \alpha, \\
\tilde{Q}\alpha &= -[c, \alpha] + \epsilon^{IJ} \epsilon^K [\phi_{Ik}, \psi_J] - \frac{1}{2} \epsilon^I D_a \eta_I, \\
\tilde{Q}\phi_{IJ} &= -[c, \phi_{IJ}] + \frac{1}{2} (\epsilon_I \eta_J + \epsilon_J \eta_I), \\
\tilde{Q}\eta_I &= -[c, \eta_I] + \epsilon^{JK} \epsilon^M [\phi_{JM}, \phi_{IK}], \\
\tilde{Q}c &= -c^2 + \epsilon^I \epsilon^J \phi_{IJ}.
\end{aligned} \tag{2.16}$$

in agreement with the transformation found in the works of [15, 14]; the nilpotence reads as

$$(\tilde{Q})^2 \propto \delta_{\phi_{IJ}}, \tag{2.17}$$

that is an infinitesimal transformation of  $\phi_{IJ}$ . With the result of the previous section, we are ready to write down the Blau-Thompson action, which is the invariant Yang-Mills action for the topological theory.

### 3 The Blau-Thompson action

The associated action for  $N_T = 2$ ,  $D = 4$  is the Witten action [8, 15, 16], described in  $N_T = 2$  by the Blau-Thompson action [13, 14], with gauge completely fixed in terms of the superfield. For the construction of this action, we wish a Lagrange multiplier that couples to the topological super-Yang-Mills so as to manifest its self-duality:  $F = *F$ . We then define a 2-form-superfield Lagrange multiplier, with the property of anti-self-duality and super-gauge covariant:  $sK = -[C, K]$ , such that

$$K(x, \theta) = k(x) + \theta^I k_I(x) + \frac{1}{2} \theta^2 \kappa(x).$$

We still wish a quadratic term in the last component field of  $K$ . Still, we need a 0-form-superfield to complete the gauge-fixing for  $\Psi_I$ , which is defined as:

$$H_I(x, \theta) = h_I(x) + \theta^J h_{JI}(x) + \frac{1}{2} \theta^2 \rho_I(x). \tag{3.1}$$

To fix the super-Yang-Mills gauge, we define an anti-ghost superfield for  $C$ , being a 0-form-superfield of fermionic nature

$$\overline{C}(x, \theta) = \overline{c}(x) + \theta^I \overline{c}_I(x) + \frac{1}{2} \theta^2 \overline{c}_F(x), \tag{3.2}$$

we define a 0-form-superfield Lagrange multiplier

$$B(x, \theta) = b(x) + \theta^I b_I(x) + \frac{1}{2} \theta^2 \beta(x). \tag{3.3}$$

Their BRST transformations are  $s\overline{C} = B$ ,  $sB = 0$ , and in components they reads

$$\begin{aligned}
s\overline{c} &= b, & s\overline{c}_I &= b_I, & s\overline{c}_F &= \beta, \\
sb &= 0, & sb_I &= 0, & s\beta &= 0.
\end{aligned} \tag{3.4}$$

Therefore the complete Blau-Thompson action in superspace takes the form

$$S_{BT} = \int d^2\theta \sqrt{g} \text{Tr} \{ K * F + \zeta K * D_\theta^2 K + \epsilon^{IJ} H_I D_A * \Psi_J + s(\overline{C} d * A) \}, \tag{3.5}$$

with  $\zeta$  being constant. In components, we have

$$\begin{aligned}
S_{BT} = & \int \sqrt{g} Tr \{ \frac{1}{2} \kappa * f + \zeta \kappa * \kappa + \zeta \varepsilon^{IJ} (k * [\eta_I, k_J] + [k_J, \eta_I] * k) - \zeta \phi^{IJ} \phi_{IJ} k * k \\
& - \frac{1}{2} \varepsilon^{IJ} k_I * D_a \psi_J + \frac{1}{2} k * D_a \alpha + \frac{1}{4} k * \varepsilon^{IJ} [\psi_I, \psi_J] \\
& + \varepsilon^{IJ} (\frac{1}{2} \rho_I D_a * \psi_J + \frac{1}{2} h_{JI} D_a * \alpha - \frac{1}{2} \varepsilon^{KL} h_{KI} D_a * D_a \phi_{JL} \\
& + \frac{1}{2} h_I D_a * D_a \eta_J - \varepsilon^{KL} h_I D_a * [\psi_K, \phi_{JL}] - \frac{1}{2} [h_I, \psi_J] * \alpha \\
& - \frac{1}{2} \varepsilon^{KL} [\psi_K, h_I] * D_a \phi_{JL} + \frac{1}{2} \varepsilon^{KL} [\psi_K, h_{LI}] * \psi_J + [\alpha, h_I] * \psi_J) \\
& + \frac{1}{2} b d * B + \frac{1}{2} \varepsilon^{IJ} b_I d * \psi_J + \frac{1}{2} \beta d * a - \frac{1}{2} \bar{c} d * D_a c_F \\
& - \frac{1}{2} \varepsilon^{IJ} \bar{c} d * [\psi_J, c_J] - \frac{1}{2} \bar{c} d * [B, c] + \frac{1}{2} \varepsilon^{IJ} \bar{c}_I d * D_a c_J \\
& + \frac{1}{2} \varepsilon^{IJ} \bar{c}_I d * [\psi_J, c] - \frac{1}{2} \bar{c}_F d * D_a c \}. \tag{3.6}
\end{aligned}$$

where  $g$  is the beckground metric of the Riemannian manifold.

In the next section, we shall discuss the Avdeev-Chizhov action in a general Riemannian manifold with the same background metric.

## 4 Tensorial Matter in a General Riemannian Manifold

To couple the theory above to the Avdeev-Chizhov model, we start describing the Avdeev-Chizhov action through the complex self-dual field  $\varphi$  [6], initially written in the 4-dimensional Minkowskian manifold, whose indices are:  $m, n, \dots$ . We write this action, according to the work of [6], as

$$S_{matter} = \int d^4x \{ (D^m \varphi_{mn})^\dagger (D_p \varphi^{pn}) + q (\varphi_{mn}^\dagger \varphi^{pn} \varphi^{\dagger mq} \varphi_{pq}) \}. \tag{4.1}$$

Here  $q$  is a coupling constant for the self-interaction, and the covariant derivative  $D_a^m \varphi_{mn} = \partial^m \varphi_{mn} - [a^m, \varphi_{mn}]$ ;  $a^m$  is the Lie-algebra-valued gauge potential and we assume  $\varphi_{mn}$  to belong a given representating of the gauge group  $G$ . This action is invariant under the folowing transformations:

$$\delta_G(\omega) a_m = D_m \omega, \quad \delta_G(\omega) \varphi_{mn} = \varphi_{mn} \omega, \quad \delta_G(\omega) \varphi_{mn}^\dagger = -\omega \varphi_{mn}^\dagger, \tag{4.2}$$

with  $\varphi$  given by

$$\varphi_{mn} = T_{mn} + i \tilde{T}_{mn}, \tag{4.3}$$

which exhibit the properties  $\varphi_{mn} = i \tilde{\varphi}_{mn}$ ,  $\tilde{\tilde{\varphi}}_{mn} = -\varphi_{mn}$ , where the duality is defined by  $\tilde{\varphi}_{mn} = \frac{1}{2} \varepsilon_{mnpq} \varphi^{pq}$ .

To treat this theory, in a general Riemannian manifold as a topological theory, Geyer-Mülsch [4] rewrite the field in a four-dimensional Riemannian manifold, endowed of the vierbein  $e_\mu^m$  and a spin-connection  $\omega_\mu^{mn}$ , i.e., the tensorial matter read as  $\varphi_{\mu\nu} = e_\mu^m e_\nu^n \varphi_{mn}$ , where the action (4.1) is given by

$$S_{matter} = \int d^4x \sqrt{g} \{ (\nabla_\mu \varphi^{\mu\nu})^\dagger (\nabla_\rho \varphi^\rho_\nu) + q (\varphi_{\mu\nu}^\dagger \varphi^{\rho\nu} \varphi^{\dagger\mu\lambda} \varphi_{\rho\lambda}) \}. \tag{4.4}$$

In this 4-dimensional Riemannian manifold, we find the folowing properties:

$$\sqrt{g} \varepsilon_{\mu\nu\rho\lambda} \varepsilon^{mnpq} = e_{[\mu}^m e_\nu^n e_\rho^p e_{\lambda]}^q, \tag{4.5}$$

$$e_\mu^m e_\nu^n g^{\mu\nu} = \eta^{mn}, \quad e_\mu^m e_\nu^n \eta_{mn} = g_{\mu\nu}. \quad (4.6)$$

The covariant derivative in the Riemannian manifold is now written in terms of the spin-connection:

$$\nabla_\mu = D_\mu + \omega_\mu, \quad (4.7)$$

where  $\omega_\mu = \frac{1}{2}\omega_\mu^{mn}\sigma_{mn}$ , being  $\sigma_{mn}$  the generator of the holonomy Euclidean group  $SO(4)$ , also we have:  $D_\mu = (D_a)_\mu$ , where,  $a$ , is the Yang-Mills connection.

## 5 Supersymmetrization of the Avdeev-Chizhov Action

From now on, we can write the action (4.4) in terms of superfields, mentioning the conventions of the works [10, 8]. The superfield that accommodates the rank-two anti-symmetric tensorial matter field, is similar to the one defined in [7], being now expressed as a linear fermionic. This is defined as a rank-two anti-symmetric tensor in the 4-dimensional Riemannian manifold, and with the topological fermionic index  $I$  referring to the topological SUSY index:

$$\Sigma_{\mu\nu}^I(x, \theta) = \lambda_{\mu\nu}^I(x) + \theta^I \varphi_{\mu\nu}(x) + \frac{1}{2}\theta^2 \zeta_{\mu\nu}^I(x), \quad (5.1)$$

where  $\varphi_{\mu\nu}(x)$  is the Avdeev-Chizhov field. The super-manifold is composed by Riemannian manifold and the  $N_T = 2$  topological manifold.

The superfield is defined under the SUSY transformations

$$Q_I \Sigma_{\mu\nu J} = \partial_I \Sigma_{\mu\nu J}, \quad (5.2)$$

and in components:

$$\begin{aligned} Q_I \lambda_{\mu\nu J} &= \varepsilon_{IJ} \varphi_{\mu\nu} \\ Q_I \varphi_{\mu\nu} &= -\zeta_{\mu\nu I} \\ Q_I \zeta_{\mu\nu J} &= 0 \end{aligned} \quad (5.3)$$

Based on the work of ref. [6], we rewrite the BRST transformations, referring the non-Abelian Avdeev-Chizhov model, in terms of the transformations:

$$\begin{aligned} s\varphi_{mn}^i &= ic^a (T^a)^{ij} \varphi_{mn}^j, & s\varphi_{mn}^{\dagger i} &= -ic^a \varphi_{mn}^{\dagger j} (T^a)^{ji}, \\ s(\nabla_m \varphi_{mn})^i &= ic^a (T^a)^{ij} (\nabla_m \varphi_{mn})^j, & s(\nabla_m \varphi_{mn})^{\dagger i} &= -ic^a (\nabla_m \varphi_{mn})^{\dagger j} (T^a)^{ji}, \end{aligned}$$

where (2.2) is the Lie algebra. We wish to write the BRST-transformation for a supergauge transformation, generalizing the transformations for the Avdeev-Chizhov fields, according to

$$\begin{aligned} s(\Sigma_{\mu\nu}^I) &= iC(\Sigma_{\mu\nu}^I), \\ s(\Sigma_{\mu\nu}^I)^\dagger &= iC(\Sigma_{\mu\nu}^I)^\dagger; \end{aligned} \quad (5.4)$$

in components, we get:

$$\begin{aligned} s\lambda_{\mu\nu}^I &= ic\lambda_{\mu\nu}^I, \\ s\lambda_{\mu\nu}^{\dagger I} &= -ic\lambda_{\mu\nu}^{\dagger I}, \\ s\varphi_{\mu\nu} &= ic\varphi_{\mu\nu} + ic^I \lambda_{\mu\nu I}, \\ s\varphi_{\mu\nu}^\dagger &= -ic\varphi_{\mu\nu}^\dagger - ic^I \lambda_{\mu\nu I}^\dagger, \\ s\zeta_{\mu\nu}^I &= ic\zeta_{\mu\nu}^I - ic^I \varphi_{\mu\nu} + ic_F \lambda_{\mu\nu}^I, \\ s\zeta_{\mu\nu}^{\dagger I} &= -ic\zeta_{\mu\nu}^{\dagger I} + ic^I \varphi_{\mu\nu}^\dagger - ic_F \lambda_{\mu\nu}^{\dagger I}. \end{aligned} \quad (5.5)$$

The super-derivative of the (5.1) is covariant under the BRST-transformation, where now, the covariant super-derivative is

$$\mathcal{D}_\mu(\cdot) = (D_A)_\mu(\cdot) + \omega_\mu(\cdot) = \nabla_\mu(\cdot) + \theta^I [\psi_{I\mu}, (\cdot)] + \frac{1}{2}\theta^2 [\alpha_\mu, (\cdot)],$$

according to (4.7), then gives

$$\begin{aligned} s(\mathcal{D}_\mu \Sigma_{\mu\nu}^I) &= C(\mathcal{D}_\mu \Sigma_{\mu\nu}^I), \\ s(D_I \Sigma_{\mu\nu}^I) &= C(D_I \Sigma_{\mu\nu}^I), \end{aligned}$$

where we chose here,  $s\omega_\mu = 0$ .

By now performing BRST-transformations on the components that survive in the  $N_T = 2$  Wess-Zumino gauge (2.15), we find:

$$\begin{aligned} \tilde{Q}\lambda_{\mu\nu I} &= \epsilon^J \varepsilon_{JI} \varphi_{\mu\nu} + ic\lambda_{\mu\nu I}, \\ \tilde{Q}\lambda_{\mu\nu I}^\dagger &= \epsilon^J \varepsilon_{JI} \varphi_{\mu\nu}^\dagger - ic\lambda_{\mu\nu I}^\dagger, \\ \tilde{Q}\varphi_{\mu\nu} &= ic\varphi_{\mu\nu} + i\epsilon^I \zeta_{\mu\nu I} + i\epsilon^I \phi_{IJ} \lambda_{\mu\nu}^J, \\ \tilde{Q}\varphi_{\mu\nu}^\dagger &= -ic\varphi_{\mu\nu}^\dagger - i\epsilon^I \zeta_{\mu\nu I}^\dagger - i\epsilon^I \phi_{IJ} \lambda_{\mu\nu}^{\dagger J}, \\ \tilde{Q}\zeta_{\mu\nu I} &= ic\zeta_{\mu\nu I} - i\epsilon^J \phi_{JI} \varphi_{\mu\nu} + i\epsilon^J \eta_J \lambda_{\mu\nu I}, \\ \tilde{Q}\zeta_{\mu\nu I}^\dagger &= -ic\zeta_{\mu\nu I}^\dagger + i\epsilon^J \phi_{JI} \varphi_{\mu\nu}^\dagger - i\epsilon^J \eta_J \lambda_{\mu\nu I}^\dagger, \end{aligned} \quad (5.6)$$

in agreement to (2.17).

We build up rank-two anti-symmetric tensorial matter field in a superspace formulation, leaving the superfield with the same properties as shown in [7]; this is invariant under gauge transformations (5.6) and SUSY transformations. The kinetic term is proposed as

$$S_{kin} = \int d^4x d^2\theta \sqrt{g} \varepsilon^{IJ} \{ (\mathcal{D}_\mu \Sigma_I^{\mu\nu})^\dagger (\mathcal{D}_\rho \Sigma_{\nu J}^\rho) \}.$$

In components, we get:

$$\begin{aligned} S_{kin} &= \int d^4x \sqrt{g} \{ \frac{1}{2} (\nabla_\mu \varphi^{\mu\nu})^\dagger (\nabla_\rho \varphi_{\nu}^\rho) + \frac{1}{2} \varepsilon^{IJ} (\nabla_\mu \lambda_I^{\mu\nu})^\dagger (\nabla_\rho \zeta_{\nu J}^\rho) \\ &\quad + \frac{1}{2} \varepsilon^{IJ} (\nabla_\mu \zeta_I^{\mu\nu})^\dagger (\nabla_\rho \lambda_{\nu J}^\rho) + (\nabla_\mu \varphi^{\mu\nu})^\dagger [\psi_\rho^I, \lambda_{\nu I}^\rho] \\ &\quad + [\psi_\mu^J, \varphi^{\dagger\mu\nu}] (\nabla_\rho \varphi_{\nu}^\rho) + \varepsilon^{IJ} (\nabla_\mu \lambda_I^{\mu\nu})^\dagger ([\alpha_\rho, \lambda_{\nu J}^\rho] + [\psi_\rho^J, \varphi_{\nu}^\rho]) \\ &\quad + \varepsilon^{IJ} ([\alpha_\mu, \lambda_I^{\dagger\mu\nu}] + [\psi_\mu^J, \varphi^{\dagger\mu\nu}]) (\nabla_\rho \lambda_{\nu J}^\rho) \} \end{aligned} \quad (5.7)$$

The interaction term has the peculiarity of presenting two derivatives of the Grassmann coordinates; it should also be invariant under the gauge transformations (5.6) and supersymmetry. We write it as

$$S_{int} = \int d^4x d^2\theta \sqrt{g} \{ \varepsilon^{IJ} \varepsilon^{LM} (\Sigma_{\mu\nu I})^\dagger D^K (\Sigma_J^{\rho\nu}) (\Sigma_L^{\mu\lambda})^\dagger D_K (\Sigma_{\rho\lambda M}) \} \quad (5.8)$$

where  $D_K(\cdot) = \partial_K(\cdot) + [E_K, (\cdot)]$ ; in components,

$$\begin{aligned} S_{int} &= \frac{1}{2} \int d^4x \sqrt{g} \{ \varphi_{\mu\nu}^\dagger \varphi^{\rho\nu} \varphi^{\dagger\mu\lambda} \varphi_{\rho\lambda} - \varepsilon^{IJ} [(\lambda_{\mu\nu I}^\dagger \zeta_J^{\rho\nu} + \zeta_{\mu\nu I}^\dagger \lambda_J^{\rho\nu}) \varphi^{\dagger\mu\lambda} \varphi_{\rho\lambda} \\ &\quad - \varphi_{\mu\nu}^\dagger \varphi^{\rho\nu} (\lambda_I^{\dagger\mu\lambda} \zeta_{\rho\lambda J} + \zeta_I^{\dagger\mu\lambda} \lambda_{\rho\lambda J})] + \varepsilon^{IJ} \varepsilon^{KL} [\lambda_{\mu\nu I}^\dagger \zeta_J^{\rho\nu} (\lambda_K^{\dagger\mu\lambda} \zeta_{\rho\lambda L} + \zeta_K^{\dagger\mu\lambda} \lambda_{\rho\lambda L}) \\ &\quad + \zeta_{\mu\nu I}^\dagger \lambda_J^{\rho\nu} (\lambda_K^{\dagger\mu\lambda} \zeta_{\rho\lambda L} + \zeta_K^{\dagger\mu\lambda} \lambda_{\rho\lambda L}) + \lambda_{\mu\nu I}^\dagger \lambda_J^{\rho\nu} [\eta_L, \lambda_K^{\mu\lambda}] \varphi_{\rho\lambda} \end{aligned} \quad (5.9)$$

$$- \lambda_{\mu\nu I}^\dagger \varphi_J^{\rho\nu} \eta_L \lambda_K^{\dagger\mu\lambda} \lambda_{\rho\lambda} + \varphi_{\mu\nu}^\dagger \lambda_J^{\rho\nu} \eta_L \lambda_K^{\dagger\mu\lambda} \lambda_{\rho\lambda L} - \lambda_{\mu\nu I}^\dagger \lambda_J^{\rho\nu} \eta_L \lambda_K^{\dagger\mu\lambda} \varphi_{\rho\lambda} \quad (5.10)$$

$$- \lambda_{\mu\nu I}^\dagger \lambda_J^{\rho\nu} \eta_K \varphi^{\dagger\mu\lambda} \lambda_{\rho\lambda L} + \lambda_{\mu\nu I}^\dagger \lambda_J^{\rho\nu} \phi^{MN} \phi_{MN} \lambda_K^{\dagger\mu\lambda} \lambda_{\rho\lambda L} \}]. \quad (5.11)$$

The total action is being determinad for:  $S_{Kin} + qS_{Int}$ , such that

$$S_{AC} = - \int d^2\theta \sqrt{g} \{ \varepsilon^{IJ} (\mathcal{D}_\mu \Sigma_I^{\mu\nu})^\dagger (\mathcal{D}_\rho \Sigma_{\nu J}^\rho) + q \varepsilon^{IJ} \varepsilon^{LM} (\Sigma_{\mu\nu I})^\dagger D^K (\Sigma_J^{\rho\nu}) (\Sigma_L^{\mu\lambda})^\dagger D_K (\Sigma_{\rho\lambda M}) \}, \quad (5.12)$$

where  $q$  is a quartic coupling constant. In components, we have the Avdeev-Chizhov action plus its partness:

$$\begin{aligned}
S_{AC} = & \int d^4x \sqrt{g} \left\{ \frac{1}{2} (\nabla_\mu \varphi^{\mu\nu})^\dagger (\nabla_\rho \varphi^\rho{}_\nu) + \frac{1}{2} \varepsilon^{IJ} (\nabla_\mu \lambda_I^{\mu\nu})^\dagger (\nabla_\rho \zeta^\rho{}_{\nu J}) \right. \\
& + \frac{1}{2} \varepsilon^{IJ} (\nabla_\mu \zeta_I^{\mu\nu})^\dagger (\nabla_\rho \lambda^\rho{}_{\nu J}) + (\nabla_\mu \varphi^{\mu\nu})^\dagger [\psi_\rho^I, \lambda^\rho{}_{\nu J}] \\
& + [\psi_\mu^J, \varphi^{\dagger\mu\nu}] (\nabla_\rho \varphi^\rho{}_\nu) + \varepsilon^{IJ} (\nabla_\mu \lambda_I^{\mu\nu})^\dagger ([\alpha_\rho, \lambda^\rho{}_{\nu J}] + [\psi_\rho^J, \varphi^\rho{}_\nu]) \\
& + \varepsilon^{IJ} ([\alpha_\mu, \lambda_I^{\dagger\mu\nu}] + [\psi_\mu^J, \varphi^{\dagger\mu\nu}]) (\nabla_\rho \lambda^\rho{}_{\nu J}) \\
& + q (\varphi_{\mu\nu}^\dagger \varphi^{\rho\nu} \varphi^{\dagger\mu\lambda} \varphi_{\rho\lambda} - \varepsilon^{IJ} [(\lambda_{\mu\nu}^\dagger \zeta_J^{\rho\nu} + \zeta_{\mu\nu}^\dagger \lambda_J^{\rho\nu}) \varphi^{\dagger\mu\lambda} \varphi_{\rho\lambda} \\
& - \varphi_{\mu\nu}^\dagger \varphi^{\rho\nu} (\lambda_I^{\dagger\mu\lambda} \zeta_{\rho\lambda J} + \zeta_I^{\dagger\mu\lambda} \lambda_{\rho\lambda J})] + \varepsilon^{IJ} \varepsilon^{KL} [\lambda_{\mu\nu}^\dagger \zeta_J^{\rho\nu} (\lambda_K^{\dagger\mu\lambda} \zeta_{\rho\lambda L} + \zeta_K^{\dagger\mu\lambda} \lambda_{\rho\lambda L}) \\
& + \zeta_{\mu\nu}^\dagger \lambda_J^{\rho\nu} (\lambda_K^{\dagger\mu\lambda} \zeta_{\rho\nu L} + \zeta_K^{\dagger\mu\lambda} \lambda_{\rho\nu L}) + \lambda_{\mu\nu}^\dagger \lambda_J^{\rho\nu} [\eta_L, \lambda_K^{\mu\lambda}] \varphi_{\rho\lambda} \\
& - \lambda_{\mu\nu}^\dagger \varphi_J^{\rho\nu} \eta_L \lambda_K^{\dagger\mu\lambda} \lambda_{\rho\lambda} + \varphi_{\mu\nu}^\dagger \lambda_J^{\rho\nu} \eta_I \lambda_K^{\dagger\mu\lambda} \lambda_{\rho\lambda L} - \lambda_{\mu\nu}^\dagger \lambda_J^{\rho\nu} \eta_L \lambda_K^{\dagger\mu\lambda} \varphi_{\rho\lambda} \\
& \left. - \lambda_{\mu\nu}^\dagger \lambda_J^{\rho\nu} \eta_K \varphi^{\dagger\mu\lambda} \lambda_{\rho\lambda L} + \lambda_{\mu\nu}^\dagger \lambda_J^{\rho\nu} \phi^{MN} \phi_{MN} \lambda_K^{\dagger\mu\lambda} \lambda_{\rho\lambda L}] \right\}. \tag{5.13}
\end{aligned}$$

It is invariant under conformal transformations. Therefore, the total gauge invariant action can be written as:  $S_{AC} + S_{BT}$ . We could also have replace  $S_{BT}$  by the super- $BF$  action described in the work of ref. [11].

The  $Q$ -exactness of the total action above is also true for  $N_T = 2$  SUSY as in [4]; this is so because the fermionic volume element  $Q^2 \propto Q_1 Q_2$ , which means the exactness in the charge  $Q_1$ ,  $Q_2$  of this action. This proof for  $N_T = 1$  and general  $N_T$ , is given in the works [10], where the total action is also  $s$ -exact. According to Blau-Thompson in their review [17], the energy-momentum tensor  $\Theta_{\mu\nu}$  is also  $Q$ -exact,

$$\mathcal{O} = \langle 0 | \Theta_{\mu\nu} | 0 \rangle = \langle 0 | \frac{2}{\sqrt{g}} \frac{\delta}{\delta g^{\mu\nu}} (S_{BT} + S_{AC}) | 0 \rangle = \langle 0 | Q \Upsilon_{\mu\nu} | 0 \rangle \tag{5.14}$$

ensuring the topological nature of the theory, where we shall just use the Avdeev-Chizhov kinetic term, because the interaction term carries the coupling constant  $q$ , which is irrelevant for the attainment of the observables of the theory [4].

## Concluding Remarks

The main goal of this paper is the settlement of a topological superspace formulation for the investigation of the coupling between the rank-two Avdeev-Chizhov matter field and Yang-Mills fields. It comes out that the stress tensor is  $Q$ -exact. This opens us the way for the identification of a whole class of observables that we are trying to classify [19].

It is worthwhile to draw the attention here to the shift symmetry that allows us to detect the ghost character of the Avdeev-Chizhov field. On the other hand, it is known that there appears a ghost mode in the spectrum of excitations of our tensor matter field [1]. The connection between these two observations remain to be clarified. The fact that the Avdeev-Chizhov field manifest itself as a ghost guide future developments in the quest for a consistent mechanism to systematically decouple the unphysical mode mentioned above.

We are also trying to embed the tensor field in the framework of a gauge theory with Lorentz symmetry breaking [18]. We expect that this breaking may identify the right ghost mode present among the two spin 1 components of the Avdeev-Chizhov field.



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## Appendices

### A Conventions

The topological fermionic index:  $I = 1, 2$ , is lowered and raised by the anti-symmetric Levi-Civita tensor:  $\varepsilon_{IJ}, \varepsilon^{IJ}$ , with  $\varepsilon^{12} = -\varepsilon_{12} = 1$ . The  $\theta$ -coordinates definitions:  $\theta^I = \varepsilon^{IJ}\theta_J$ ,  $\theta_I = \varepsilon_{IJ}\theta^J$ , the quadratic forms are:

$$\theta^2 = \theta^I \theta_I = -\theta_I \theta^I, \quad \theta^I \theta^J = -\frac{1}{2} \varepsilon^{IJ} \theta^2, \quad \theta_I \theta_J = \frac{1}{2} \varepsilon_{IJ} \theta^2,$$

with  $\varepsilon_{IK} \varepsilon^{KJ} = \delta_I^J$ . The derivatives in the  $\theta$ -coordinates are defined by

$$\partial_I = \frac{\partial}{\partial \theta^I}, \quad \partial^I = \frac{\partial}{\partial \theta_I} \quad \text{and} \quad \partial_I \theta^J \stackrel{Def}{=} \delta_I^J. \quad (\text{A.1})$$

thus we have

$$\partial_I f(x, \theta) = \varepsilon_{IJ} \partial^J f(x, \theta),$$

with  $f(x, \theta)$  a any superfunction. Deriving the  $\theta$ -coordinates gives

$$\partial^I \theta^J = -\varepsilon^{IJ}, \quad \partial_I \theta_J = -\varepsilon_{IJ} \quad (\text{A.2})$$

A superfield is expanded as:  $F(x, \theta) = f(x) + \theta^I f_I(x) + \frac{1}{2} \theta^2 f_F$ , obeying the transformation  $Q_I F(x, \theta) \stackrel{Def}{=} \partial_I F(x, \theta)$ . In components, we have:

$$Q_I f = f_I; \quad Q_I f_J = -\varepsilon_{IJ} f_F; \quad Q_I f_F = 0. \quad (\text{A.3})$$

Characteristics table of the superconnection fields:

Charge \ fields	$\epsilon^I$	$a$	$\psi^I$	$\alpha$	$\chi^I$	$\phi^{IJ}$	$\eta^I$	$c$	$c^I$	$c_F$
$s$	-1	0	1	2	1	2	3	0	1	2
$g$	1	0	0	0	0	0	0	1	1	1
$p$	0	1	1	1	0	0	0	0	0	0
$P_{grs}$	+	-	+	-	-	+	-	-	+	-

(A.4)

where  $s$ : susy number,  $g$ : ghost number,  $p$ : degree form,  $P_{grs}$ : Grassmann parity.

### B Rules for Topological Grassmannian integration

The definition of integration in this topological SUSY representation is

$$\int d\theta^I \stackrel{Def}{=} \partial_I. \quad (\text{B.1})$$

This result is applied to a superfunction  $f(x, \theta)$ , so that the volume element is

$$\int d^2\theta f(x, \theta) \stackrel{Def}{=} \frac{1}{4} \varepsilon^{IJ} \partial_I \partial_J f(x, \theta); \quad (\text{B.2})$$

therefore, the square of the supersymmetric charge operator (shift operator) is defined by:

$$Q^2 = Q^I Q_I = \partial^I \partial_I = 4 \int d^2\theta,$$

which is a volume element too.

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